

## Parameterization of frozen ground effects: sensitivity to soil properties

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**Abstract** Heat and moisture transfer processes in the aeration zone play an important role in the runoff generation mechanism in regions where seasonal soil freezing/thawing occurs. Seasonally frozen soil can significantly influence the amount of runoff generated during winter and spring. This study presents an analysis of a physically-based parameterization of the frozen ground effects derived from Kozeny's theory that accounts for changes in both volumetric liquid water and solid particles–water contact surface. An analytical solution is formulated for the Sacramento soil moisture accounting model linked with a basic heat transfer model. Tests at selected sites and river basins show that simulated soil moisture, temperature, and runoff agree well with measured data. Simulation results at river basin outlets suggest that a non-frozen version consistently underestimates spring floods and overestimates the following summer floods if the frozen ground effect is significant. It is impossible to remove these biases without introducing frozen ground physics. Solid particles–water contact surface change is the dominant factor in the runoff mechanism affected by frozen ground; however, its effect decreases significantly for the coarser soils.

**Key words** floods; frozen ground; prediction; soil temperature and moisture; USA

### INTRODUCTION

Heat and moisture transfer processes in the aeration zone play an important role in the runoff generation mechanism in regions where seasonal soil freezing/thawing occurs. Seasonally frozen soil can significantly influence the amount of runoff generated during winter and spring. Lack of vegetation, shallow snow cover, and cold temperature are ideal conditions for the formation of deep frost. Recent developments in land surface modelling have significantly improved representation of cold season processes (e.g. Cherkauer & Lettenmaier, 1999; Koren *et al.*, 1999). However, a conceptual representation of a soil profile in commonly used watershed models complicates implementation of physically-based heat–moisture transfer models that require numerical integration over the soil profile. Another challenge is the formulation of the effects of frozen ground on water fluxes, specifically the partitioning of meltwater/rainfall into surface runoff and infiltration. In watershed modelling, simplified approaches are typically used such as empirical equations of the percolation reduction (Koren, 1980; Anderson & Neuman, 1984) or water balance-type approaches (Cherkauer & Lettenmaier, 1999). The latter assumes the percolation reduction depends only on frozen water-induced soil porosity reduction. However, field and experimental data suggest that an increase in the surface contact between solid particles and soil water may also be a factor.

This study addresses two of the problems mentioned above: (a) modification of a storage-type rainfall–runoff hydrological model to be compatible with the theoretical heat transfer model, and (b) parameterization of frozen ground effects on runoff derived from Kozeny’s theory (Kulik, 1969) that accounts for changes in both solid particles–water surface contact and “free” porosity. Frozen ground effect analysis is performed with a lumped version of the Sacramento Soil Moisture Accounting (SAC-SMA) model on four river basins in Minnesota, USA.

## PARAMETERIZATION OF FROZEN GROUND PROCESSES

### Combining heat-moisture transfer components

The SAC-SMA model is used to estimate soil moisture states and runoff components, and a layer integrated form of the heat transfer model (Koren *et al.*, 1999) is used to estimate soil temperature and unfrozen water states. The SAC-SMA model consists of upper and lower tension and free water storages that interact to generate five runoff components. Koren *et al.* (2002) have developed a set of relationships that link the SAC-SMA storages (parameters) and soil properties such as porosity, field capacity, wilting point, and hydraulic conductivity. They assume that tension water storages of the SAC-SMA model are related to available soil water, and that free water storages are related to gravitational soil water.

The model parameter-soil property relationships allow recalculating the upper and lower soil moisture capacities into soil moisture contents at a number of soil layers. Five layer depths are defined *a priori* to cover a 2 m soil profile with thinner layers closer to the soil surface. However, an actual number of soil layers and their thicknesses are automatically adjusted using SAC-SMA parameter values at the selected location. To make this adjustment, the upper,  $Z_U$ , and lower,  $Z_L$ , zone depths are estimated first to be sure that the upper and lower SAC-SMA capacities are preserved (for the SAC-SMA model schematic see Fig. 1):

$$Z_U = \frac{UZTWM + UZFWM}{\theta_s - \theta_{wt}} \quad (1)$$

$$Z_L = Z_U + \frac{LZTWM + LZFSM + LZFPM}{\theta_s - \theta_{wt}} \quad (2)$$

where  $UZTWM$ ,  $UZFWM$ ,  $LZTWM$ ,  $LZFSM$ , and  $LZFPM$  are SAC-SMA parameters: upper zone tension and free water, and lower zone tension water and supplemental and primary free water storages respectively,  $\theta_s$  and  $\theta_{wt}$  are saturation soil moisture content and wilting point respectively. *a priori* defined layer depths are then adjusted to be consistent with the  $Z_U$  and  $Z_L$  estimates. Because of this, the number of soil layers may be less than five, and can be different for different pixels. A variable number of soil layers can be used in the heat transfer model (Koren *et al.*, 1999) simplifying the coupling to the SAC-SMA model. To be compatible with SAC-SMA complexity, e.g. the use of precipitation and air temperature input data only, the soil/snow surface heat balance calculation of the original heat transfer model is replaced by applying air

temperature as a soil surface temperature. At each time step, SAC-SMA liquid water storage changes due to snowmelt/rainfall are estimated, and then they are transformed into the layered soil moisture states of the heat transfer model. The heat transfer model splits the total water content into frozen and liquid water portions based on simulated soil temperature profile. Newly estimated soil moisture states are then converted back into the SAC-SMA model storages using the same relationships between SAC-SMA storage parameters and soil properties as described above.

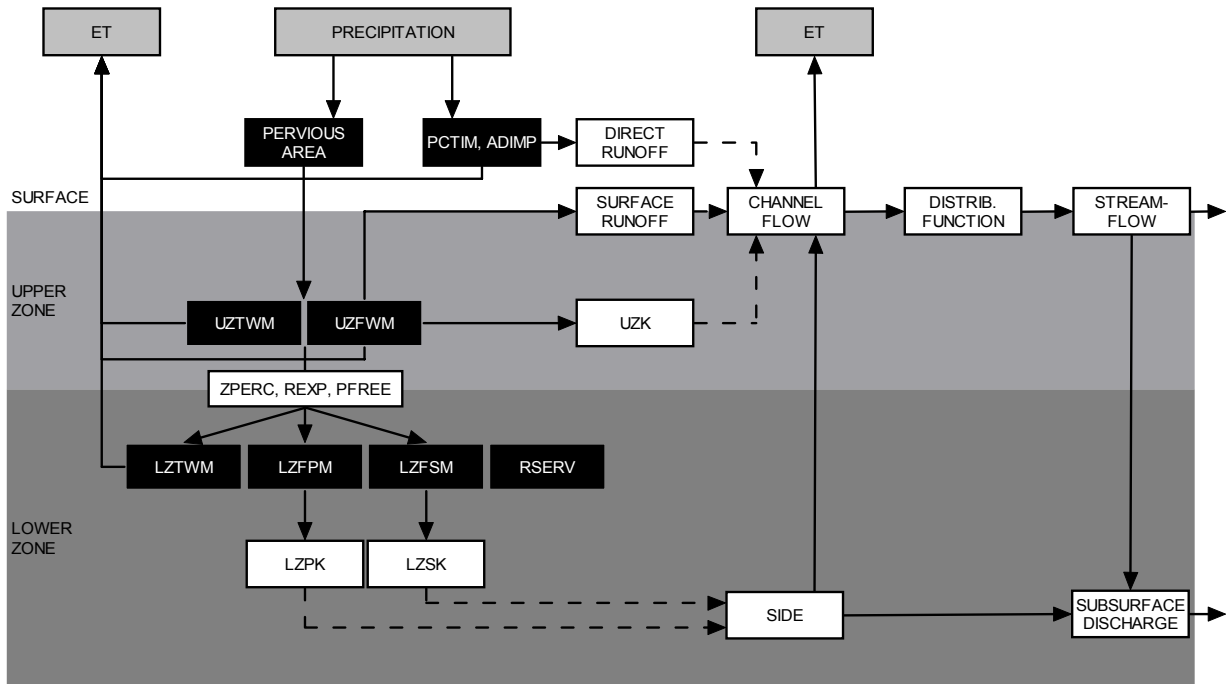


Fig. 1 Schematic of the SAC-SMA model.

### Formulation of water fluxes due to frozen ground effects

The most common formulation of water fluxes under frozen ground conditions assumes that reduction of soil moisture conductivity depends only on the ratio of a liquid water content ( $\theta_l$ ) to the soil saturation ( $\theta_s$ ), e.g.:

$$K_f = K_0 f\left(\frac{\theta_l}{\theta_s}\right) \quad (3)$$

In this formulation, liquid water content is used instead of total water content but the saturated hydraulic conductivity,  $K_0$ , is assumed to be the same value as for the unfrozen soil. However, field and experimental data suggest that an increase in the contact surface between solid particles and soil water is a bigger factor. To account for this additional frozen ground effect on the saturated hydraulic conductivity  $K_0$ , we use Kozeny's theory that relates the filtration rate and the soil particles–water contact surface,  $S_0$ :

$$K_0 = A \frac{P^3}{S_0^2} \quad (4)$$

where  $P$  is the soil porosity, and  $A$  is a parameter that depends on soil properties.

Following Kulik (1969), we assume that increase in the contact surface due to ice crystals equals  $a\theta_f$ , where  $a$  is an increase in the ice crystal surface per unit of ice content, and  $\theta_f$  is the ice content. By including this additional contact surface in equation (4) and dividing the result by the original equation (4), one can obtain an analytical formulation of this effect on the saturated hydraulic conductivity of the frozen ground:

$$K_0^* = K_0 \frac{1}{[1 + (a/S_0)\theta_f]^2} \quad (5)$$

and

$$K_f = K_0^* f\left(\frac{\theta_l}{\theta_s}\right) \quad (6)$$

It can be seen from equations (5) and (6), that hydraulic conductivity can be reduced significantly due to change in the surface contact in addition to the liquid water ratio. Experimental data suggest that the value of  $a/S_0$  does not vary significantly for different soils, and a value of eight is its reasonable estimate.

The original SAC-SMA model runoff components are formulated as linear reservoirs with constant release rates which can be expressed as a linear function of the hydraulic conductivity. Therefore, to account for the frozen ground effect, the ratio of the hydraulic conductivity from equation (5) to the saturated hydraulic conductivity  $K_0$  is applied to the interflow and fast groundwater runoff components as well as to percolation to the lower zone. It is assumed that the slow groundwater component from a deeper soil layer is not affected by this factor directly.

## RESULTS AND DISCUSSION

### Soil temperature and soil moisture simulation results

Results of earlier tests (Koren *et al.*, 1999) of the heat transfer model coupled with a simple water balance model (Schaake *et al.*, 1999) have shown good agreement of simulated and observed soil temperature and soil moisture for two experimental data sets. Here, we have extended tests for a number of operational stations over the northwestern part of the USA with available measurements of precipitation, air temperature, and soil temperature (from NCDC cooperative network databases). At most stations, soil temperatures were measured at multiple depths, 5, 10, 20, 50, and 100 cm once per day, usually between 07:00 h and 12:00 noon. Hu & Feng (2003) reported that these measurements well represented daily mean temperature at depths below 50 cm (root mean square error is less than 0.25°C) while accuracy reduced to 1.3°C at 5 cm soil layer. The modified version of the original heat transfer model coupled with the SAC-SMA model (as described in the previous section) is used in

these tests. Model parameters are estimated from soil-vegetation data (Koren *et al.*, 2002) without further calibration. Daily frozen and liquid water contents and soil temperature at five layers are simulated for 3–5 years.

Overall simulated soil temperature dynamics are represented well for all five soil layers. Correlation coefficients between simulated and measured soil temperatures are above 0.95 while there is a slight decrease in correlation for the top layer; see Table 1 for statistics at three selected soil layers. Some reduction in variability of the top layer soil temperature can be explained using daily input data. Root mean square errors and Nash-Sutcliffe efficiency (Nash & Sutcliffe, 1970) are also better for the deeper soil layer.

### Test results for selected river basins

Four river basins in Minnesota, USA have been selected for runoff simulation tests. Selected basins and their areas are listed in Table 2. Six-hourly precipitation and air temperature averaged over each basin and daily discharges at basin outlets are available for a 25–40 year period. Empirical unit hydrographs are used to transform basin average simulated runoff into outlet hydrographs. First, we calibrate SAC-SMA parameters of frozen and non-frozen ground versions using 10 years of data. Note, that the frozen ground version does not have any additional parameters to calibrate. The remaining 15–30 years of data are used for test purposes.

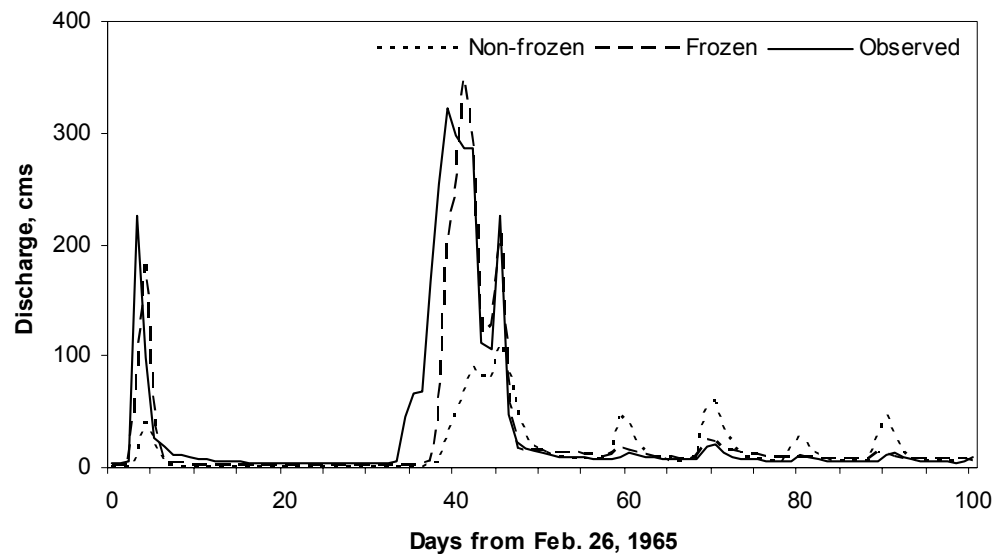
**Table 1** Accuracy statistics of simulated soil temperature at three layers: root mean square error (RMS), Nash-Sutcliffe efficiency (NS), and correlation coefficient (R).

| Station name    | NCDC    | 5 cm layer |      |      | 20 cm layer |      |      | 50 cm layer |      |      |
|-----------------|---------|------------|------|------|-------------|------|------|-------------|------|------|
|                 | Site ID | RMS        | NS   | R    | RMS         | NS   | R    | RMS         | NS   | R    |
| Lamoni, IA      | 134585  | 3.0        | 0.91 | 0.96 | 2.9         | 0.90 | 0.99 | 2.9         | 0.88 | 0.99 |
| Atlantic, IA    | 130364  | 5.0        | 0.77 | 0.96 | 3.3         | 0.86 | 0.99 | 2.9         | 0.82 | 0.98 |
| Burlington, IA  | 131060  | 3.3        | 0.84 | 0.96 | 2.5         | 0.88 | 0.97 | 3.0         | 0.79 | 0.97 |
| Des Moines, IA  | 132209  | 3.7        | 0.89 | 0.97 | 1.9         | 0.95 | 0.99 | 1.6         | 0.95 | 0.99 |
| Estherville, IA | 132724  | 4.5        | 0.83 | 0.96 | 2.0         | 0.95 | 0.98 | 2.6         | 0.88 | 0.98 |
| Toledo, IA      | 138296  | 3.2        | 0.89 | 0.97 | 3.2         | 0.88 | 0.98 | 2.3         | 0.91 | 0.99 |
| Preston, MN     | 216654  | 3.9        | 0.87 | 0.97 | 2.4         | 0.91 | 0.98 | 2.4         | 0.87 | 0.99 |
| Waubay, SD      | 398980  | 3.3        | 0.90 | 0.96 | 1.5         | 0.97 | 0.99 | 1.3         | 0.97 | 0.99 |
| Spencer, IA     | 137844  | 8.1        | 0.67 | 0.95 | 5.1         | 0.80 | 0.98 | 3.6         | 0.85 | 0.99 |
| Mean            |         | 4.2        | 0.84 | 0.96 | 2.8         | 0.90 | 0.98 | 2.5         | 0.88 | 0.99 |

**Table 2** Root mean square errors of daily (DRMS) and monthly (MRMS) discharges from frozen and non-frozen versions.

| Basin ID | Basin area (km <sup>2</sup> ) | With frozen ground |           | Without frozen ground |           |
|----------|-------------------------------|--------------------|-----------|-----------------------|-----------|
|          |                               | DRMS (cm)          | MRMS (cm) | DRMS (cm)             | MRMS (cm) |
| LNEM5    | 1593                          | 13.91              | 3.85      | 16.14                 | 5.98      |
| GRDM5    | 2204                          | 9.95               | 5.48      | 11.20                 | 6.36      |
| MMLM5    | 671                           | 3.40               | 1.59      | 3.90                  | 1.98      |
| HWYM5    | 842                           | 2.67               | 1.66      | 2.92                  | 1.68      |

Root mean square errors of daily and monthly discharges for the test period are shown in Table 2. The frozen ground version outperforms the non-frozen ground version for all basins. Comparison of simulation results from frozen and non-frozen ground versions for the Root River (LNEM5 in Table 2) is shown in Fig. 2. Spring floods are significantly underestimated and early summer floods are overestimated from the non-frozen version. These results suggest that the model parameter calibration can not fix the non-frozen ground version physics problem.



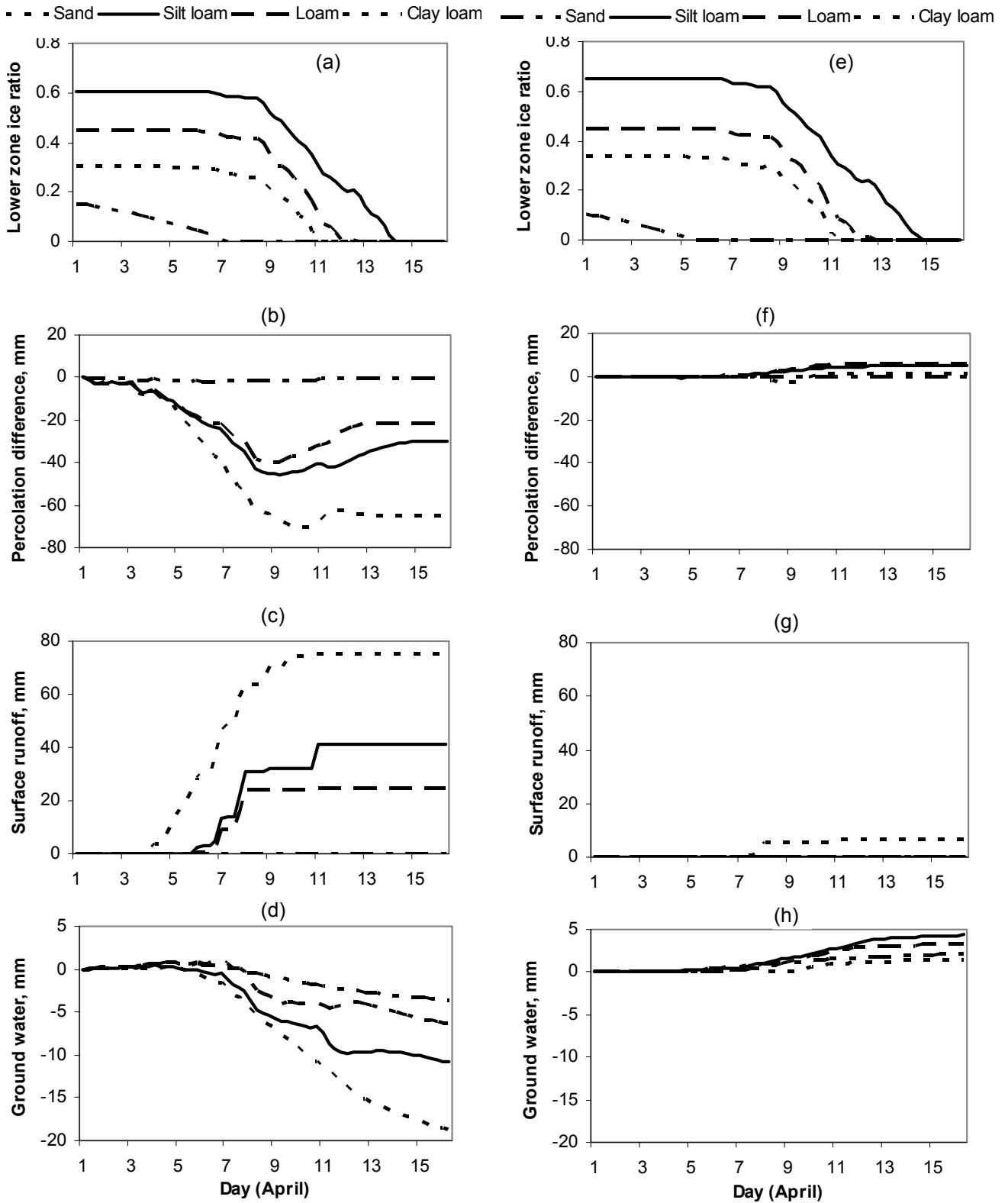
**Fig. 2** Observed hydrograph and simulated hydrographs from frozen and non-frozen versions for the Root River, Minnesota, USA.

### Soil property effects on water balance components due to frozen ground

To analyse the effects of volumetric and solid particles–water contact surface changes due to frozen ground conditions, we have performed sensitivity tests with different soil types. For each soil type, *a priori* parameters are estimated without further calibration. The Root River (LNEM5) input and output data are used in these tests.

Simulation results are shown in Fig. 3. It can be seen that the soil type significantly affects the percolation and runoff components. Sand type soils are practically not affected by the frozen ground even under very wet conditions. Less penetrated finer soils with a significant clay fraction are affected most of all. For all soils, percolation and groundwater runoff decreases and surface runoff increases due to frozen soil conditions. Comparison of Fig. 3(a–d) (left panels) and Fig. 3(e–h) (right panels) suggests that under similar frozen ground conditions (close ice-liquid water ratios, Fig. 3(a) and (e)) solid particles–water contact surface is a much bigger factor than just volumetric liquid water changes.

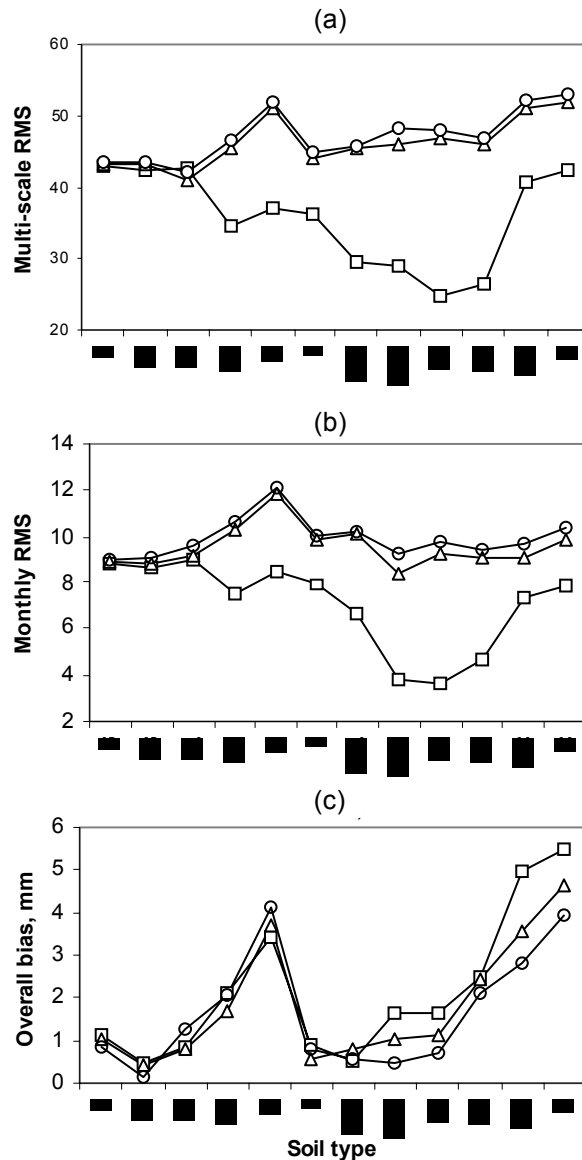
Another sensitivity test has been performed for the Root River. The same input fluxes are applied assuming different soil texture from very light sandy soil (S) to heavy clay soil (C). As in previous tests, we use soil-based *a priori* model parameters without any adjustment. Simulations are conducted for the non-frozen model version,



**Fig. 3** Comparison of ice ratio and cumulative differences of percolation, surface, and ground runoffs simulated for different soils using different parameterization of the hydraulic conductivity: (a–d) equation (6) vs equation (3), (e–h) equation (3) vs non-frozen ground version.

and two options of the frozen version, with and without effect of solid particles–water contact surface, equations (6) and (3), respectively. Accuracy statistics are calculated for one hydrological year most affected by frozen ground.

A multi-scale error that consists of root mean squared errors of daily, 3-day, 10-day, and 60-day averaged runoffs as well as root mean squared errors of monthly runoff, and overall biases for different assumed soil types are shown in Fig. 4(a–c). Multi-scale and monthly errors vary significantly for different soils due to frozen ground, with lowest errors for the clay loam soil (CL). However, runoff biases that cover spring and summer floods are changed slightly preserving overall water balance for all soil types. We can also observe that the effect of a volumetric liquid water change due to frozen soil is minimal for all statistics compared to the effect of a solid particles–water contact surface change.



**Fig. 4** Effect of frozen ground on simulation results for different soil types from non-frozen ground model (circles), and frozen ground model with different parameterization of the hydraulic conductivity: (squares) using equation (6), and (triangles) using equation (3).



## SUMMARY

A conceptual representation of soil moisture fluxes combined with a heat transfer model provides reasonable simulations of water and heat exchange over a soil profile. Nash-Sutcliffe efficiencies for simulated soil temperature at three layers for tested gauged sites exceed 0.67 with correlation coefficients above 0.95. The frozen ground parameterization does not introduce additional parameters to calibrate.

The frozen ground version out-performs a non-frozen version for all tested basins. Spring–summer flood analysis suggests that it is impossible to remove runoff biases consistently by rainfall–runoff model parameter adjustments without introducing frozen ground physics.

The physically-based modification of the hydraulic conductivity due to frozen ground accounts for changes in both volumetric liquid water and solid particles–water contact surface. Accounting for the solid particles–water contact surface change, which is more critical than the change in liquid water content, results in more accurate simulations of soil moisture and runoff.

Further analysis of the models is considered as an extension of an on-going nationwide application of the distributed hydrological modelling system. This activity will speed up evaluation and operational use of the model over a variety of soils and climates.

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